Introduction and objectives

Supply chain management is an integrative approach for planning and control of materials and information flows with suppliers and customers as well as between different functions within a company. One particularly important element in supply chain management is the management of inventories. This field has received considerable attention during the last years. Nevertheless, most quantitative analysis on supply chain management issues is dominated by the framework of multi-echelon serial systems or distribution systems where relationships between a single vendor and a single buyer or a single vendor and several buyers are considered. The situation of multiple suppliers and a single/multiple buyers has received less attention. (Minner, 2003)

Although strategies like Just-In-Time and Total-Quality-Management often suggest that a manufacturer use a single supplier in order to build a long-term supplier relationship to improve the service quality, using multiple suppliers is still very popular in practice. According to a report by McMillan, Toyota and Honda had one supplier for 28 and 38%, respectively; another 39 and 44% had two suppliers, and the rest had three or more suppliers. US government defense agencies are mandated to maintain more than one source for all but very small procurements. In 2000, a fire in a Phillips semiconductor plant in Albuquerque created a shortage of radio frequency chips for two of its buyers, Nokia and Ericsson. While Nokia managed the crisis successfully by working with its alternative suppliers, Ericsson lost at least $400 million in potential revenue since Phillips was its only source for these chips (see Mishra and Tadikamalla (2006)).

Multiple sourcing exists when several, distinctly independent sources are available for a good or a service and the buying company purchases these goods or services from more than one supplier. Single sourcing, on the other hand, is opting to have one supplier for long-term relationships, although other adequate suppliers may be available and a choice is possible.

The decision concerning single and multiple sourcing entails the decision about the number of suppliers with whom a purchasing company wants to have supplier relationships for a certain good or service, or possibly for a segment of goods or services.
Table 1 summarizes the managerial issues discussed in the literature in favor of single sourcing and multiple sourcing.

Table 1. Factors developed in the literature in favor of single and multiple sourcing (Mishra & Tadikamalla (2006))

<table>
<thead>
<tr>
<th>Single sourcing is favored</th>
<th>Multiple sourcing is favored</th>
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<tbody>
<tr>
<td>Cost</td>
<td>Switching costs remain low</td>
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<tr>
<td>High cost of close cooperation</td>
<td>No supplier has unfair advantage in negotiations</td>
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<td>Higher trust between supplier and buyer</td>
<td>Low cost and high performance through competitive bidding</td>
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<td>Economies of scale and learning curve advantages for supplier</td>
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<tr>
<td>Lower cost due to quantity discounts</td>
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<tr>
<td>High setup/order cost</td>
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<tr>
<td>Production</td>
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<tr>
<td>Better quality due to long-term relationship and associated</td>
<td>High risk of disruption of supply due to fire, strike, natural</td>
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<tr>
<td>investments</td>
<td>disaster, financial insolvency, etc.</td>
</tr>
<tr>
<td>Better understanding of product and process specifications</td>
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<tr>
<td>Quality control easier due to one source of variation</td>
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<tr>
<td>Facilitates Just-in-Time</td>
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<tr>
<td>Lower uncertainty of demand for supplier</td>
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<tr>
<td>Competitiveness</td>
<td>Access to new and wider variety of technology from among which</td>
</tr>
<tr>
<td>Superior quality improves competitiveness; poor quality of a</td>
<td>commitment can be made to the most suitable technology</td>
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<tr>
<td>supplier can reduce market share and competitiveness</td>
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In this overall title there are several dimensions to deal with. Some of these dimensions are listed below. We must point out that this list shouldn’t be considered as a classification of inventory control under multiple sourcing.

Lot splitting

One subject that has attracted many researchers recently is pooling lead time risk by splitting replenishment orders among multiple suppliers. Here the cornerstone is that all possible suppliers have stochastic replenishment lead times. Therefore when deciding for a replenishment (triggered by for instance a (s,Q) policy) it can be of advantage to split the replenishment order into several sub replenishment orders. The idea is to protect one against the randomness in the lead-times. Pooling lead-time uncertainty among several suppliers is a way to reduce the safety stock needed to meet service targets or alternatively, the expected number of backorders for a prescribed level of safety stock. Furthermore, successive deliveries of smaller orders will reduce cycle stock.

Emergency orders

Here there is a base-supplier who most often is used. However in cases where the inventory level is too low and the first arriving coming replenishment of the base-supplier is still far distant, it may be of advantage to issue an emergency order (probably more expensive but also faster). This phenomenon is coined emergency ordering.

Exploiting price differentials
A specific realisation of this could be the following. Assume the inventory system is in Denmark. There is one potential supplier in Denmark and another in, say, Mexico. The unit purchase price of the Danish supplier is constant while the unit purchase price of the Mexican supplier fluctuates (due to the Mexican peso fluctuates compared to the Danish krone). Here the decision from when and from where to source will depend on the current inventory level as well as relationship between the two unit purchase costs.

**Uncertain supply capacity**

Most models assume that the supply capacity is unlimited and some assume there is supply capacity but it is known. However, unexpected machine breakdowns could affect the supply capacity. Relative little amount of work has been done in the area of uncertain supply capacity. For example Parlar and Perry (1996) present a continuous time model in which the availability of each of the n suppliers is on or off. For each of these \(2^n\) states, they analyze a \((r,Q)\) ordering policy.

Another problem which can be put in this dimension is supply disruption: suppose that there are two suppliers that have the capacity to provide a needed critical part for the manufacturer’s final product. Supplier 1 is located outside the manufacturer’s country, and offers competitive price; however, this supplier is prone to breakdown or disruption in supply or even the supplied material can experience loss because of significant lead time. On the other hand, Supplier 2 is local, reliable but more expensive. The problem is to decide which policy is preferred: single sourcing, where supplier 1 is the only source for that critical part, or dual sourcing. The difference between this subject and emergency orders is that here there is a probability of breakdown or disruption in supply for the main supplier.

Among these dimensions for multiple sourcing I defined some projects for my PhD thesis to work on. The following is the list and some brief explanation about each project. If we want to categorize these projects, we can say that project A and B are about the first above-mentioned dimension; project C1 is about the second above-mentioned dimension, project C2 is about both first and second above mentioned dimensions, and finally project D is about the first and second dimension.

**Description of projects defined for PhD thesis:**

**A. Determining optimal suppliers under stochastic lead times.**

The policy of simultaneously splitting replenishment orders among several suppliers has received considerable attention in the last few years and continues to attract the attention of researchers.

Sculli and Wu (1981) appear to be the first to present the concept of order splitting. They analyzed the two supplier situation with normally distributed lead times and used numerical integration to derive tables for the mean and standard deviation of the lead time demand. Their study, as well as that of others, indicated that a firm can achieve higher
service levels for any level of safety stock by effectively distributing lead-time over several suppliers rather than only one individual supplier (See Thomas and Tyworth (2006)).

Sculli and Shum (1990) extended the model for more than two suppliers with non-identical allocation portions of supply and gave expressions for the mean and variance of the effective lead times.

In order to assess the benefits of order splitting in an economic context it is necessary to consider models where the total cost for ordering, purchase prices, inventory holding, and stock out penalties are minimized. Ramasesh et al. (1991) seem to be the first to present such a cost minimization approach for two potential suppliers. In their model, suppliers are assumed to be of the same reliability, so the order quantity is evenly split between the suppliers. Both suppliers have identical lead time distributions, being either uniform or exponential. Demand is assumed to be constant, shortages are backordered, and a penalty cost per item per time unit is incurred. From the non-linear total cost functions, the optimal values for the reorder point s and the order quantity Q are delivered by numerical search. In Ramasesh et al. (1993), these findings are extended to the case of suppliers with non-identical lead time distributions, purchasing prices, and order splitting portions. In their model, both suppliers are assumed to have exponential lead time distributions.

Chiang and Benton (1994) showed that dual sourcing is better than single sourcing except for the situation where the order cost is high and lead time variability is low.

One of the most important models of order splitting in an economic context is the model of Sedarage et al. (1999). Their model is not restricted to only two suppliers. Lead times of suppliers and demand arrival are probabilistic and the unit purchasing prices from different suppliers may be different, thus, the order quantities for different suppliers may vary as well. They developed a mathematical model to determine the reorder level and the order quantity for each supplier so that the expected total cost per time unit, consisting of the fixed ordering cost, procurement cost, inventory holding cost, and shortage cost, is minimized. Tyworth and Ruiz-Torros (2000) considered transportation cost in their model and found that it can strongly influence the single versus dual sourcing decision under a broad set of realistic conditions. Their assumptions are similar to Ramasesh et al. (1993).

In general, some authors such as Sculli and Wu (1981), Hayya et al. (1987), Kelle and Silver (1990a, 1990b), Sculli and Shum (1990), Fong et al. (2000), and Kelle and Miller (2001) have focused on statistical theories and methods for estimating the effect of splitting on the distribution function, mean, and variance of effective lead times. On the other hand, many authors have concentrated on economic analysis. They have developed long run average cost models to assess the performance of split models in relation to non-split models under common conditions. Most of these models include inventory holding cost, ordering cost, and shortage cost.

A major weakness in the above studies is that almost all of them only consider order splitting when there are only two suppliers, i.e. dual sourcing. It means that they neglect the effect of order splitting on the system when there are more than two suppliers. Most of the authors have concluded that dual sourcing in many cases is better than one sourcing, but are these results true when orders are split between more than two suppliers, or in general, n suppliers?
The case that we consider in this paper is developing a mathematical model which considers multiple-supplier single-item inventory systems.

In this paper, we develop an analytical model for determining optimal order splitting and reorder level for n-supplier inventory systems. This project was done by Soheil, supervised by Dr. Farahani, before his start as a PhD student. The only comparable model in the literature is that of Sedarage et al. (1999), which is an approximate model. To find the advantages of our model with regard to their model, we compare both models with that of Ramasesh et al. (1993), which is an exact model for dual sourcing assuming no crossover among orders. As our extensive numerical experiments show, Sedarage et al. (1999) model always underestimate the total cost compared with Ramasesh et al.’s (1993) model. On the other hand, the results from our developed model always (in the case of dual sourcing) coincide with that of Ramasesh et al. (1993).

The item acquisition lead times of suppliers are random variables. Backorder is allowed and shortage cost is charged based on not only per unit in shortage but also per time unit. We have developed two models: one for constant demand rate and the other one for probabilistic demand rate but all our comparisons between models and numerical studies are based on constant demand rate. Continuous review (s, Q) policy has been assumed. When the inventory level depletes to a reorder level, the total order is split among n suppliers. Since the suppliers have different characteristics, the quantity ordered to different suppliers may be different.

The problem is to determine the reorder level and quantity ordered to each supplier so that the expected total cost per time unit, including ordering cost, procurement cost, inventory holding cost, and shortage cost, is minimized.

This project is finished.

B. Dual sourcing for decaying products:

Until now, all researches about order splitting were under the assumption that our products are perfect and they don’t deteriorate over time. But in reality many products may lose their values over time.

According to Goyal and Giri (2001), inventoried goods can be broadly classified into three meta-categories based on

(a) Obsolescence,
(b) Deterioration,
(c) No obsolescence/deterioration.

Obsolescence refers to items that lose their value through time because of rapid changes of technology or the introduction of a new product by a competitor.
Deterioration refers to the damage, spoilage, dryness, vaporization, etc. of the products. Products like foodstuffs, green vegetables, human blood, etc., having a maximum usable lifetime are known as perishable products and the products like alcohol, gasoline, radioactive substances, etc. having no shelf-life at all are known as decaying products.

In this project, we will consider the ordering policy for decaying products when we split orders between suppliers having stochastic lead times. Certain commodities shrink with time by a proportion which can be approximated by a negative exponential function of time. This observation led to the modeling of the inventory items with decay processes by the differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -D,$$

where $\theta$ is the constant decay rate, $I(t)$ the inventory level at time $t$, and $D$ is the demand rate. Our model is under the assumption of continues review $(s, Q)$ policy. The order is placed when the reorder level is reached and can be split (not necessarily equally) between the two suppliers. Backorder is allowed and shortage cost is charged per product unit for shortage per time unit. The suppliers lead times are random variables $Y_1$ and $Y_2$. We will consider two distribution functions for them: uniform and exponential. This two distribution functions are very good for investigating the effect of lead time variability on the system performance measures when we use order splitting. In our model we assume that the demand rate is constant and time horizon is infinite. In addition, decaying products are not replaced.

In numerical analyses section we are going to design some experiments to see the effect of changing different variable on the system and costs and then we intend to compare these results with those of single sourcing system to see the relative benefits of dual sourcing in the presence of decaying goods. For instance a working hypothesis could be to demonstrate that as $\theta$ increase the relative benefits of dual source system compared to single source system increases.

Until now first we developed an approximate model, but then we developed it to an exact model. The model is almost finished.

C. Incorporating both order split and emergency ordering:

The main premise for studying inventory control models with several suppliers is that the lead-times of the suppliers are stochastic, hence it make sense to split any replenishment order into several smaller orders, in order to hedge against the risk of possible long lead-times. In those dual/multiple source models, it is always assumed that when deciding for replenishment, then the replenishment order is split into suborders and the suborders are issued simultaneously to the suppliers. In this section we let this assumption about simultaneousness be relaxed. We define two subprojects in this subject. The methodology in each subproject is different. In the first project we use nonlinear programming for modelling the inventory system and in the second one we use markov decision process.

C1. Which one is better: Dual sourcing and ordering to both suppliers simultaneously or using the second supplier as emergency source:
As was mentioned before dual or multiple sourcing with order splitting has been proved that in the case of randomness in lead-times is preferred to single sourcing model in many situations. This is because we are pooling lead-time risks between suppliers. Most researches in the literature have compared between dual sourcing and single sourcing.

In this project we develop a mathematical model for a two-supplier single item inventory system. The retailer uses one supplier as the main source and the second one as emergency source. This is our emergency model which we developed it in the case of random lead-times. In order to be able to compare this emergency ordering system with order splitting system existing in the literature, we apply the same conditions on the developing emergency ordering model: we assume that demand rate is constant and doesn’t change over time. All unsatisfied demands are backlogged; both suppliers have random lead times, but of course the lead time mean and variance for emergency supplier is less than the normal supplier. On the other hand ordering from emergency supplier is more expensive than from normal one. Continuous review \((s_n, s_e, Q_n, Q_e)\) policy has been assumed. It means that the main order \(Q_n\) is placed whenever the inventory position reaches the normal reorder level \(S_n\). If the order doesn’t arrive in proper time and inventory level reaches \(s_e\) \((s_e < s_n)\) then the emergency order \(Q_e\) is triggered. The time horizon in our model is infinite and we are interested about \((s_n^*, s_e^*, Q_n^*, Q_e^*)\) which minimizes the expected total cost.

The conditions for order splitting model is like the ones defined in project A, but only for two suppliers So we don’t mention them here again.

One question which may be asked is that which model works better from minimizing costs in the system point of view. The advantage of order splitting model is that the ordering cost is less than emergency model when you order to both suppliers in one cycle. On the other hand in emergency model you have this option that only order to the main supplier, so the ordering cost will be less than the first model. In this paper we want to compare between the two models in different situations.

After developing the model, by designing many experiments we are going to compare between this model and dual sourcing with order splitting. We would like to know under which situation which model is better to implement and saves system costs more.

Some of the system parameters that we are interesting to see their effects on each model in our comparison are:

\(h\): unit holding cost per time unit

\(\pi\) : unit shortage cost per time unit

\(K_1, K_2,\) and \(K\): ordering cost per order for the two suppliers.

\(C_n, C_e\): unit purchase cost from normal supplier and emergency supplier

\(\lambda\): lead time parameter
At the moment the model for emergency orders is finalized and we are in the middle of programming for solving the model.

**C2. Is it optimal to make replenishments simultaneously in a dual source model**

In this project we study a dual source system with non-identical suppliers. We still are using most the assumptions used in the standard model but model the problem as a semi-Markov decision model, allowing the decision maker the choice whether he will simultaneously issue two orders to both suppliers or he will first issue a single order to one supplier and then await further information concerning his inventory status before making a replenishment to the other supplier (the latter having some resemblance to an emergency order system). In our cost structure we have joint replenishment cost, thus making it of some advantage to do joint ordering. So a main issue for investigation in our numerical analysis is to explore how large this cost figure must be in order to make it in-optimal to deviate from making replenishments simultaneously.

The demand is a Poisson process with intensity $\lambda$. We assume that there is a fixed order size $Q_j$, $j=1, 2$ when making a replenishment to supplier $j$. Furthermore, we assume that only one order can be outstanding at each supplier. The lead-time of supplier $j$ is assumed to be exponentially distributed with mean $1/\mu_j$, $j=1, 2$. If the replenishment orders are issued simultaneously a replenishment cost of $K + K_1 + K_2$ is incurred. If only a replenishment order to supplier $j$ ($j = 1, 2$) is incurred then the replenishment cost $K_j$. Thus, the cost parameter $K$ is a sort of joint order cost and if it is large compared to the two supplier-specific order costs $K_1$ and $K_2$ then there is an incentive to make replenishments simultaneously. The inventory system is incurred inventory costs at a rate $h$ per unit. All demand that cannot be met immediately is lost and incurs a cost $\pi$ per unit.

We develop a semi-markov decision model to find the optimal policy by using the value iteration algorithm. The algorithm to compute an optimal $(s_1, s_2, c_1, c_2, Q_1, Q_2)$ policy consists of two main loops: an inner loop, with a given $Q_1$ and $Q_2$, where for a given order size to each supplier we use dynamic programming value iteration on a truncated state space (by limiting the amount of inventory in the system) to find the optimal $(s_1, s_2, c_1, c_2)$ policy, and an outer main loop where $Q_1$ and $Q_2$ are varied over a sufficiently large interval.

In addition to this optimal policy we introduced a heuristic method in order to find near-optimal policy.

Then we developed the main model for the case of Erlang lead time distributions which is more realistic and general than exponential distribution. For this developed model, we assume that the lead-time of Supplier $j$ is an $R_j$-phased Erlang distributed with mean $R_j/\mu_j$. In our Markov chain model we assume the phases of the supply processes are observable. This is probably not a reasonable assumption. Therefore a main focus of our investigation is how to modify the optimal policy, assuming observable phases, to a policy that does not assume that the phases of the supply processes can be observed, and we are interested to know how much the costs of the policy increase by this assumption.
Until now, the modelling for this project is finished and we did programming and solved the two models for some preliminary test problems.

As is seen, both subprojects C1 and C2 are about the same topic, but the assumptions are different in each of them: deterministic demand (constant demand rate) in C1 and random demand (Poisson process and Erlang distribution) in C2. So therefore one interesting research question could be to explore the impact of the demand assumption on the conclusion when to shift from splitting to emergency ordering.

D. Investigating the impact of supply information in a dual source inventory system

This project deals with the influence of having some information about the supply lead time on ordering. In this project we introduce two scenarios of having information about lead times and then try to compare between these scenarios.

We consider dual source inventory model with two different suppliers. One supplier, Supplier 1, has a lead-time that is dependent on his current backlog of orders while the other, Supplier 2, is assumed to have a constant lead-time ($L^2 \geq 1$). In the first scenario, Scenario 1, the inventory system is able to monitor the backlog at Supplier 1. In technical terms the inventory system knows the random process (a Markov chain) that describes how the backlog evolves over time. Concerning Supplier 1 we apply the assumption of Zipkin (2000; p 275) that the orders we submit are “virtual orders”. Therefore when modelling the backlog process of Supplier 1 we only need to consider the inflow per time unit from other customers and the output rate per time unit of Supplier1’s production system. We assume that the output rate (when the production system is in working mode because of a positive backlog) is constant $1$ unit. Therefore if the inflow can be characterized by a non-negative integer-valued random variable $X$ and backlog at the start of period $t$ is a non-negative integer $W_t$, the backlog evolves over time according to the equation (we use the notation $b^+ = \max\{b, 0\}$):

$$W_{t+1} = (W_t - 1)^+ + X$$

Therefore if we decide to submit an order when observing a backlog $w$ we will experience a lead-time that is $L^1 = \max\{w, 1\}$. Thus even if we submit an order to an “empty” Supplier 1 we assume a lead-time (for instance it may include transportation) of $1$ time unit. When in the beginning of time period $t$ issuing an order to Supplier $j$ we receive the order in the beginning of time period $t + L^j, j=1, 2$.

In the second scenario, Scenario 2, the inventory system cannot monitor this process, but just perceives the backlog at Supplier 1 as a random variable (in order to make a fair comparison we assume that its probability distribution is equal to the steady state probability distribution of the corresponding Markov chain). That is, it is first after submitting a replenishment order to Supplier 1 it knows the actual lead-time of this order. For both scenarios we develop a Markov decision model to compute the optimal inventory control policy.
We assume that the order sizes we can submit are constant (maybe required so by the suppliers) $Q_j, j=1, 2$. Furthermore, at each supplier there can at most be one replenishment order residing. The demand observed in each time period is given by a non-negative integer-valued random variable $D$ (we assume all demands are independent and identically distributed). We assume that all demand that can not immediately be filled is lost at a cost of $\pi$ per unit. All items on hand at the inventory at the end of a period are charged a cost $h$ per unit. Concerning replenishment costs, we assume if the replenishment orders are issued simultaneously a replenishment cost of $K + K_1 + K_2$ is incurred. If only a replenishment order to supplier $j (j = 1, 2)$ is issued then the replenishment cost $K + K_j$ is incurred.

After making our models for the two scenarios we will be able to compare between them. We try to identify under which situations (wrt. choice of parameters in our model) it is most beneficial to be able to monitor the backlog process at Supplier1 and in which cases it is of less importance. Besides, we are interested to know how the models split orders between two suppliers in each scenario.

At the moment, we are in first steps of this project. We have made a model for each scenario. Our future work in this project is programming and solving the models using an algorithm like value iteration algorithm and then we need to compare the results of each model in different situations.
References


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